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OBSERVATIONS OF ATMOSPHERIC
REFRACTION

1905-09

BY

H. G. SHAW,

SURVEY OF INDIA.

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Observations of Atmospheric Refraction

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1. These observations were undertaken in order to investigate the laws of atmospheric refraction. My programme was to determine:—

- (a) The refraction co-efficient both towards the plains and towards the snows.
- (b) The refraction co-efficient in the case of a great depression like the angle from Mussooree to Dehra Dun.
- (c) The hourly variation in refraction.
- (d) The seasonal variation in refraction between October and May.

My observations have already been discussed by Colonel Burrard in the preface to Synoptical Volume XXXV (North-East Longitudinal Series) and in appendices 5 and 8 of G. T. S. Professional Volume No. XIX (Levelling of Precision in India, 1858-1909). In Synoptical Volume XXXV the observations were used to determine the most probable values of heights of snow-peaks. In Professional Volume XIX my observed angles were placed on record for the information of future observers. In this paper I am deducing and classifying the several values of refraction co-efficients obtained, and in some cases also the difference of heights deduced trigonometrically and from spirit-levelling.

If C = the arc contained between the normals at the stations in seconds

D_a = depression of B from A in seconds

E_b = Elevation of A from B „

then omitting minor corrections such as those for height of instrument and of signal

$$\text{the co-efficient of refraction} = \frac{1}{2} \left\{ \frac{C - (D_a - E_b)}{C} \right\}.$$

When reciprocal angles have been observed the difference of height between stations is h where

$$h = c' \sin \frac{D_a + E_b}{2} \sec E_b.$$

If only one angle *e.g.*, E_a has been observed the other D_b is equal to $c + E_a - 2r$, where $r = Kc''$, K being the co-efficient of refraction.

2. I observed from five stations, three of which were specially built for this work.

(i). Mussooree (Shaw's Station), on the Lesser Himalayan range, height 6929·9* feet.

It is 25 feet east, and $17\frac{1}{2}$ feet south or 30 feet S.E. of Mussooree Dome Observatory Station (Evelyn Hall) (*vide* Synoptical Volume of the Great Arc Meridional Series—Section 24° to 30°, page 86*—A). It is a masonry pillar 3 feet in diameter and 1 foot 9 inches in height, upper surface being flush with ground level, with an annulus 4 inches in width. A slab bearing the inscription "Shaw's Refraction Station 1905-09" has been fixed flush with the upper surface of the pillar.

(ii). Dehra Dun (Shaw's Station), at the foot of the Lesser Himalayan range, height 2234·3* feet.

It is 10 feet west and $21\frac{1}{2}$ feet south of the Dehra Dun Latitude Station (near Haig Observatory) (*vide* G. T. S. Vol. XVIII page 59). It is 79 feet 8 inches east of the Dehra Dun Longitude Station (*vide* G. T. S. Vol. XV, Appendix page 5 and Vol. XVIII, page 59). It is a masonry pillar 3 feet in diameter and 2 feet 6 inches in height, the upper surface being flush with ground level, with an annulus 3 inches in width. A slab bearing the inscription "Shaw's Refraction Station 1905-09" has been fixed flush with the upper surface of the pillar.

(iii). Nojli (Shaw's Station), situated in the plains, height 886·7* feet.

It is 7 feet east and 19 feet south or 20 feet S.E. of Colonel Campbell's Latitude Station, 63 feet from S.E. corner and 68 feet from N.E. corner of Nojli Tower Station. It is a masonry pillar 3 feet in diameter and $2\frac{1}{4}$ feet in height, the upper surface of which is flush with ground level, with an annulus 3 inches in width. A stone 6 inches square with the letters \bigcirc inscribed on it has been embedded in the centre of the pillar, the upper surface of which is on a level with the top of the pillar.

(iv). Nojli Tower Station, upper mark, close to No. (iii), height 937·0* feet.

It is a principal station of the Great Arc Meridional Series—Section 24° to 30°—and is situated about half a mile S. of the village of the same name and about 3 miles S.E. of Balia-kheri Railway Station, O. and R. Railway.

(v). Nag Tiba Hill Station, on the range immediately beyond that on which Mussooree stands, height 9912 feet.

* Spirit-levelled height,

A circle and dot inscribed on the upper surface of a stone embedded centrally in a circular masonry pillar 40 inches in diameter and 2 feet in height. Vertically below this upper mark, 2 other stones similarly inscribed are embedded in the centre and in the foundation of the pillar respectively. Around this pillar, a circular masonry ring has been built 1 foot thick and of the same height as the pillar leaving a concentric space 3 inches wide, and surrounding this a platform of earth and stones 14 feet square has been built. It is on the summit of the highest of three peaks locally known as Nag Tiba. The hill can be approached from Mussooree in 2 marches by several different paths, all of which are very steep in places. There is a site for good camping ground on the spur running north from the station, and water is found on the north side of the main ridge to the east.

Two stations were purposely introduced at Nojli, one 50 feet higher than the other, so as to compare the effects of atmospheric refraction at ground level with those at an altitude of 50 feet above the ground.

3. The observing stations to the snow peaks were Mussooree, Nojli and Nag Tiba, and from these stations observations were taken periodically to the snow peaks of Bandarpunch, height 20720 feet, Srikanta height 20120 feet, Jaonli height 21760 feet and Kedarnath height 22770 feet. These heights are the ones deduced from the old observations and are in no way based on the observations under discussion.

4. The trigonometrical observations consisted of the following series:—

Observations of vertical angles.

- | | | |
|----------------|-----|---|
| First Series | ... | From Mussooree to Dehra Dun and <i>vice versa</i> —distance 9·454 miles. |
| Second Series | ... | From Mussooree to Nojli (Shaw's) and <i>vice versa</i> —distance 45·927 miles. |
| Third Series | ... | From Mussooree to Nojli T.S. and <i>vice versa</i> —distance 45·931 miles. |
| Fourth Series | ... | From Mussooree to Nag Tiba and <i>vice versa</i> —distance 9·886 miles. |
| Fifth Series | ... | From Mussooree to the Himalayan peaks of Bandarpunch (distance 47·129 miles), Srikanta (distance 55·474 miles), Jaonli (distance 54·062 miles) and Kedarnath (distance 63·759 miles). |
| Sixth Series | ... | From Nojli (Shaw's) to the Himalayan peaks of Bandarpunch (distance 92·923 miles), Srikanta (distance 99·800 miles), Jaonli (distance 96·961 miles) and Kedarnath (distance 104·148 miles). |
| Seventh Series | ... | From Nag Tiba to the Himalayan peaks of Bandarpunch (distance 37·370 miles), Srikanta (distance 46·495 miles), Jaonli (distance 45·814 miles) and Kedarnath (distance 56·438 miles). |

5. Results of the First Series of Observations.

Reciprocal Observations between Mussooree and Dehra Dun.

Table A.—showing *firstly* the excess in feet of the trigonometrical determination of the height of Mussooree above Dehra Dun over the spirit-level determination, and *secondly* the co-efficient of refraction deduced from the trigonometrical observations.

	Trigonometrical minus levelled height					Deduced co-efficient of refraction				
	8 a. m.	10 a. m.	Noon	2-3 p.m., time of minimum refraction	4-30 p.m.	8 a. m.	10 a. m.	Noon	2-3 p.m., time of minimum refraction	4-30 p.m.
Oct., Novr. 1905	feet 3·0	feet 2·8	feet 2·8	feet 2·8	feet 3·2	0·079	0·065	0·059	0·057	0·066
Mar., April 1906	4·3	3·2	2·9	3·0	3·2	0·076	0·066	0·057	0·056	0·068
Oct., Novr. 1906	3·2	2·7	2·6	2·7	3·2	0·070	0·060	0·054	0·055	0·058
Mar., April 1907	3·7	3·1	3·1	3·1	3·3	0·075	0·065	0·058	0·056	0·059
Oct., Novr. 1907	3·8	2·9	2·8	2·9	3·0	0·071	0·061	0·053	0·052	0·056
Mar., April 1908	3·2	2·8	2·6	2·6	3·0	0·076	0·063	0·057	0·057	0·060
October 1908	3·1	2·5	2·4	2·5	2·8	0·082	0·071	0·067	0·066	0·072
Mar., April 1909	3·8	2·8	2·9	2·8	3·0	0·079	0·066	0·062	0·062	0·067
Mean	3·5	2·9	2·8	2·8	3·1	0·076	0·065	0·058	0·058	0·063

It will be seen from *Table A* that the autumn observations give practically the same results as the spring observations, and that the maximum diurnal difference is 0·7 of a foot in height and 0·018 in the refraction co-efficient.

The following extract has been taken from Colonel Burrard's Appendix to G. T. Survey Volume XIX.

One curious fact brought to light by Mr. Shaw's observations between Dehra Dun and Mussooree is that the trigonometrical difference of height, determined from reciprocal vertical angles, is systematically larger than the spirit-level value of the difference. The orthometric difference of height determined by spirit-levelling is 4695.6 feet (=1431.2 metres): the trigonometrical determination always exceeds 4698.0 feet (=1431.9 metres).

Table showing excess in feet of the trigonometrical determination of the height of Mussooree above Dehra Dun over the spirit-level determination.

	October Novr. 1905	March April 1906	October Novr. 1906	March April 1907	October Novr. 1907	March April 1908	October 1908	March April 1909	Mean
	<i>feet</i>	<i>feet</i>	<i>feet</i>	<i>feet</i>	<i>feet</i>	<i>feet</i>	<i>feet</i>	<i>feet</i>	<i>feet</i>
8 a. m.	3.0	4.3	3.2	3.7	3.8	3.2	3.1	3.8	3.5
10 a. m.	2.8	3.2	2.7	3.1	2.9	2.8	2.5	2.8	2.9
Noon	2.8	2.9	2.6	3.1	2.8	2.6	2.4	2.9	2.8
3 p. m., time of minimum refraction	2.8	3.0	2.7	3.1	2.9	2.6	2.5	2.8	2.8
4-30 p. m.	3.2	3.2	3.2	3.3	3.0	3.0	2.8	3.0	3.1

The mean excess of the trigonometrical determination is 3.0 feet (= 0.9 metre).

I do not think that the difference between the trigonometrical and spirit-level values can be attributed to errors of observation. As the above table shows, trigonometrical observations have been carried out for years at different times of day and during different seasons, and have always given accordant results, whatever instrument has been employed. If any one is unwilling to accept the levelling results, he should study pages 342, 359 and 395 of this volume, and he will realise how unjustifiable it would be to attribute an error of even one foot to the levelling.

The diurnal variation of the difference is small, and seems to denote that the difference is not a phenomenon of refraction. Detailed calculations support this view, and show that refraction is not the primary cause. Let us, for instance, suppose that Mussooree is raised by refraction, when viewed from Dehra Dun, through x feet; and that Dehra Dun is raised by refraction, when viewed from Mussooree, through y feet. Then the discrepancy between the trigonometrical and spirit-level values will be $\frac{x-y}{2}$ feet. We have therefore to put $\frac{x-y}{2}$ equal to 3 feet, and $x-y = 6$ feet (=1.8 metres).

Now let r_1 be the co-efficient of refraction at Dehra Dun and r_2 at Mussooree; then from Mr. Shaw's observations we have the mean refraction = $\frac{r_1 + r_2}{2} = 0.056$. And we have to assume that the difference ($r_1 - r_2$) is such, that Mussooree is raised by refraction 6 feet higher when viewed from Dehra Dun, than Dehra Dun is raised, when viewed from Mussooree. We can then deduce

$$\begin{aligned} r_1 &= 0.081, \\ r_2 &= 0.031. \end{aligned}$$

But these values are contrary to all experience; and in the face of numerous independent determinations of refraction we should not be justified in accepting such a large co-efficient as 0.081 for Dehra Dun or such a small co-efficient as 0.031 for Mussooree.

The effects of refraction upon the levelling between Dehra Dun and Mussooree may be neglected; in Appendix No. 4 Mr. Hunter has shown that the total correction for refraction to spirit-levelling between Dehra Dun and Mussooree is only -0.092 foot.

The discrepancy between the trigonometrical and spirit-level values of height cannot be attributed to our employment of erroneous values of g in the formulæ for deriving the orthometric corrections for spirit-level heights, (page 100). In Appendix No. 4 of this volume, Mr. Hunter has shown that if observed values of g , (corrected for height only), are used for the deduction of the orthometric correction from the formulæ of chapter VII, the observed value of the spirit-level height of Mussooree above Dehra Dun will receive a correction of only -0.322 foot.

Great differences of opinion at present prevail amongst geologists and geodesists on the subject of the Bouguer term in the reduction of pendulum observations. Professor Suess has recently expressed distrust of the results of pendulum observations, and of the theory of isostasy*. Mr. Tittmann and Mr. Hayford have recently questioned the correctness of our orographical corrections for pendulum observations, and have put forward a theory of complete isostasy†.

But I do not think that our neglect of the Bouguer term in the deduction of the orthometric correction for spirit-levelled heights (*see* page 101) can be the cause of the discrepancy between the trigonometrical and spirit-level determinations of the Himalayan difference of height. The following are the several values of gravity:—

	at Dehra Dun	at Mussooree (Camel's Back)	difference (Dehra Dun— Mussooree)
Observed values	cm. 979·063	cm. 978·793	+ 0·270
Observed values corrected for height only	979·273	979·442	— 0·169
Observed values corrected for height, and for mass (Bouguer) ...	979·194	979·199	— 0·005
Observed values corrected for height, and for mass (Bouguer) and for orography	979·198	979·225	— 0·027
Theoretical values from Helmert's formula	979·324	979·335	— 0·011

It is difficult to see how any theory of isostatic compensation can so modify the adopted values of g , as to bring about an additional correction of + 3 feet to the spirit-levelled height of Mussooree‡. If observed values of gravity are used, without any correction at all for height or mass, the correction to the spirit-levelled height will be less than + 0·7 foot.

The only explanation that I can offer of the discrepancy of 3 feet between the trigonometrical and spirit-level values of the height of Mussooree above Dehra Dun is the following:—

(i.) The plumb-line is deflected at Dehra Dun about 30" towards the north: the observed angle of elevation of Mussooree from Dehra Dun is thus about 30" too small§.

(ii.) The plumb-line is deflected at Mussooree also about 30" towards the north; and the observed angle of depression of Dehra Dun from Mussooree is thus also 30" too small.

(iii.) Both the back and forward vertical angles being 30" too small, and the horizontal distance between Dehra Dun and Mussooree being 9·5 miles, the trigonometrical value of the difference of height has been given about 7 feet too small.

(iv.) But the spirit-level value of this difference of height is 3 feet smaller than the trigonometrical value, and as the latter is 7 feet too small, the spirit-level value must be 10 feet too small.

(v.) Calculations of trigonometrical heights are based upon a spheroidal figure for the earth; the spirit-levelling follows the surface of the geoid. The surface of the geoid must be, I think, 10 feet higher at Mussooree above the spheroidal surface than it is at Dehra Dun. This separation of the two surfaces would affect the levelling to the extent of 10 feet.

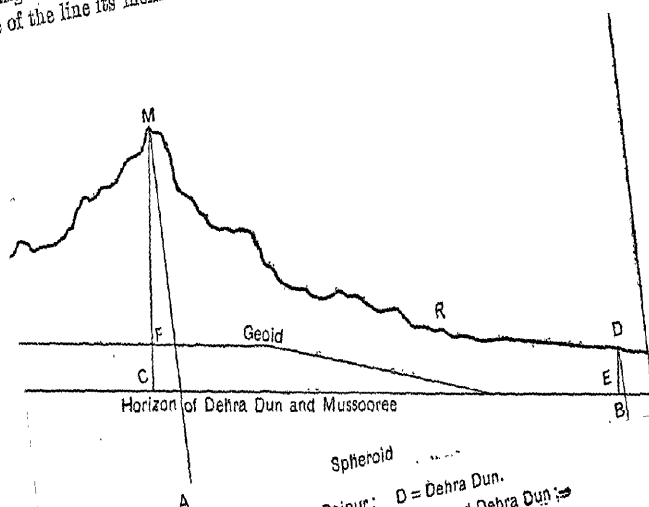
* *Vide, The face of the Earth*, Volume IV, page 610, Sollas's translation of *Das Antlitz der Erde*.

† *Report Coast and Geodetic Survey, U. S. A., 1909. Report International Geodetic Conference, Budapest, 1906. Report International Geodetic Conference, London, 1909.*

‡ The mountain line of levelling from Dehra Dun to Mussooree is the only line in India, upon which the neglect of the Bouguer term might affect the orthometric values of height in the first place of decimals of a foot.

§ For deflections see *Philosophical Transactions of the Royal Society, Vol. 205, 1905, p. 308.*

(vi.) If the deflection of the plumb-line is $30''$ both at Mussooree and Dehra Dun, it may be argued that this represents the inclination of the geoidal to the spheroidal surface, and in 9.5 miles this inclination would only produce a separation of 7 feet. This argument would be correct, if the deflection of the plumb-line *continued to be* $30''$ *throughout the line* from Dehra Dun to Mussooree. But at Rajpur, midway between Dehra Dun and Mussooree, the deflection of the plumb-line is $10''$ greater than at Dehra Dun or Mussooree. The surface of the geoid is therefore not rising at an uniform angle of $30''$ throughout the line. At the two extremities of the line it is inclined $30''$ to the spheroid; at the centre of the line its inclination exceeds $40''$.*



M = Mussooree; R = Rajpur; D = Dehra Dun.
 Difference of height between Mussooree and Dehra Dun;
 Along Spheroid $H_1 = MA - DB$.
 Along Geoid $H_2 = MC - DE$.
 And the difference of height = $H_3 = MF + DE$.
 By spirit levelling
 $H_1 - H_2 = + 7$ feet.
 $H_1 - H_3 = + 10$ "
 $H_2 - H_3 = + 3$ "

* The adopted deflections of the plumb-line are those, which have been determined in the plane of the meridian. The line joining Dehra Dun and Mussooree is some 6 degrees in azimuth east of north. In the vertical plane, passing through Dehra Dun and Mussooree, the deflections are probably $2''$ greater at both places than in the meridional plane. The course moreover of the spirit-levelling does not follow the straight line, but zig-zags to the east and to the west of it. These considerations prevent us from claiming any high degree of accuracy for our estimate of the departure of the geoidal from the spheroidal surface, but they do not affect the general principles underlying the explanation. See preface to *Synoptical Volume XXXV*, G. T. Survey of India.

6. Results of the Second Series of Observations.

Reciprocal Observations between Mussooree and Nojli (Shaw's).

Table B.—showing *firstly* the excess in feet of the trigonometrical determination of the height of Mussooree above Nojli (Shaw's Station) over the spirit-level determination, and *secondly* the co-efficient of refraction deduced from the trigonometrical observations.

	Trigonometrical <i>minus</i> levelled height					Deduced co-efficient of refraction				
	8 a. m.	10 a. m.	Noon	2-3 p. m., time of minimum refraction	4-30 p. m.	8 a. m.	10 a. m.	Noon	2-3 p. m., time of minimum refraction	4-30 p. m.
Novr., Dec. 1905	<i>feet</i> 74·7	<i>feet</i> 27·7	<i>feet</i> 6·9	<i>feet</i> 4·6	<i>feet</i> No observation	0·103	0·084	0·074	0·070	No observation
Mar., April 1906	41·8	21·9	13·1	9·8	17·5	0·087	0·073	0·068	0·067	0·070
Oct., Dec. 1906	66·4	15·1	1·9	2·3	9·2	0·098	0·081	0·072	0·069	0·072
Mar., April 1907	36·6	8·5	5·1	6·9	15·6	0·087	0·075	0·068	0·067	0·070
Mar., April 1909	48·4	20·4	4·1	3·5	7·7	0·089	0·075	0·068	0·066	0·068
Mean	53·6	18·7	6·2	5·4	12·5	0·093	0·078	0·070	0·068	0·070

In this table we find that the diurnal range both in the height and in the refraction co-efficient is great, that it is greater in autumn than in spring, that it rapidly changes between 8 a. m. and noon, and that the best time of observation is when the refraction is at minimum (about 2 p. m. to 3 p. m.). The refraction is high in the mornings and goes on decreasing to a minimum at 2 p. m., and then starts rising again between 3 and 4 p. m.

We may also learn from these results that reciprocal angles should not only be taken at the same time of day, but at the same time of year, otherwise if an observer takes the forward vertical angles at the commencement of the field season, and his reciprocals at the end of the season, some five or six months later, he will get quite a different height from what would result if all the angles were observed at nearly the same season.

It will also be noticed that in some instances, the refraction co-efficient has not changed, yet the height has done so, *e.g.*, 2-3 p. m., observations for March-April 1906, and 1907, and noon observations for March-April 1906, 1907 and 1909. The following will illustrate my meaning:—

If both D_a and E_b increase or decrease numerically by equal amounts, K which depends on $D_a - E_b$ will not be affected, but h which depends on $D_a + E_b$ will change,

e.g., Spring 1906, noon $\frac{D_a + E_b}{2} = 1^\circ 25' 49''$ and $\frac{D_a - E_b}{2} = 0^\circ 17' 16''$

Spring 1907, noon $\frac{D_a + E_b}{2} = 1^\circ 25' 42''$ and $\frac{D_a - E_b}{2} = 0^\circ 17' 16''$

The explanation is that owing probably to atmospheric changes, refraction was not the same at both places for these times and periods. In the formulæ for the computation of trigonometrical heights, the refraction at both places is assumed to be equal, but practically this assumption is often faulty. In the introduction to Synoptical Volume II, Great Arc Series 1874, Mr. W. H. Cole, M.A., writes:—

"The triangulation connecting the two base-lines was completed by 8th June 1837 as to the horizontal angles. But "some of the vertical angles remained to be observed from Aring northward; they were measured by Lieutenant Wagh "Lieutenant Jones, Mr. Logan and Mr. James, and the instruments supplied them for the purpose had vertical circles "12 inches to 18 inches in diameter. The observations were made simultaneously with the expectation that they would "be equally affected by refraction, but it was soon found that the laws of terrestrial refraction were as yet far from be- "ing discovered. The subtended angle, which is half the difference of a pair of reciprocal vertical angles, should, if the "refraction at both stations were the same, remain a practically constant quantity, however much the refraction may "vary. Far from this being the case however, the subtended angles in many instances were frequently found to vary "between limits far exceeding those of the errors of observations, thus showing that the reciprocal vertical angles were not "equally affected by refraction though observed simultaneously."

Another fact, which has also been noted by the old observers, is that refraction varies at times for the same hour on different days; if we examine the daily results of the observations at the time of minimum refraction to the snow peaks during the period under review, either during autumn or spring, we find this to be the case, the range in some instances being as much as 23 feet.

The variations of refraction are probably mainly due to the varying density of the atmosphere, and to the layer of heated air next the earth.

Table C.—showing *firstly* the differences in excess in feet of the trigonometrical determination of the height of Mussooree above Nojli (Shaw's Station) over the spirit-level determination between the autumn and spring at time of minimum refraction (2 to 3 p.m.), and *secondly* the co-efficient of refraction deduced from the trigonometrical observations.

Trigonometrical minus levelled height					Deduced co-efficient of refraction				
Year	Autumn October December	Spring March April	Autumn minus Spring	Mean of Autumn and Spring	Year	Autumn October December	Spring March April	Autumn minus Spring	Mean of Autumn and Spring
1905-06	<i>feet</i> 4.6	<i>feet</i> 9.8	<i>feet</i> — 5.2	<i>feet</i> 7.2	1905-06	0.070	0.067	+0.003	0.069
1906-07	2.3	6.9	— 4.6	4.6	1906-07	0.069	0.067	+0.002	0.068
Mean	3.5	8.4	— 4.9	5.9	Mean	0.070	0.067	+0.003	0.069

This table shows us that refraction and the difference in height vary between autumn and spring, and that the values obtained in autumn are nearer the spirit-level values than those obtained in spring, and that the mean excess of the trigonometrical value over the spirit-level value is 6 feet.

7. Results of the Third Series of Observations.

Reciprocal Observations between Mussooree and Nojli T.S. upper mark.

Table D.—showing *firstly* the excess in feet of the trigonometrical determination of the height of Mussooree above Nojli T.S. (upper mark) over the spirit-level determination, and *secondly* the co-efficient of refraction deduced from the trigonometrical observations.

	Trigonometrical <i>minus</i> levelled height					Deduced co-efficient of refraction				
	8 a. m.	10 a. m.	Noon	2-3 p.m., time of minimum refraction	4-30 p.m.	8 a. m.	10 a. m.	Noon	2-3 p.m., time of minimum refraction	4-30 p.m.
Mar., April 1906	<i>feet</i> 39·3	<i>feet</i> 20·1	<i>feet</i> 11·8	<i>feet</i> 11·8	<i>feet</i> 12·0	0·085	0·072	0·068	0·067	0·067
Mar., April 1907	42·9	16·9	9·5	10·5	11·3	0·087	0·077	0·069	0·068	0·068
Mar., April 1909	55·2	23·9	10·4	7·7	9·8	0·093	0·077	0·070	0·069	0·069
Mean	45·8	20·3	10·6	10·0	11·0	0·088	0·075	0·069	0·068	0·068

If we compare the results of the above with those corresponding to the same times and periods in *Table B*, we see that the refraction co-efficient is practically identical, but that the excess of the trigonometrical determination over the spirit-level value is greater, notwithstanding that this station is 50 feet higher than the ground level station. It was expected that the results would have been better at the higher than at the lower station.

8. Results of the Fourth Series of Observations.

Reciprocal Observations between Mussooree and Nag Tiba h.s.

Table E.—showing *firstly* the trigonometrical determination of the height of Nag Tiba h. s. above Mussooree, and *secondly* the co-efficient of refraction deduced from the trigonometrical observations.

	Trigonometrical height above Mussooree					Deduced co-efficient of refraction				
	8 a. m.	10 a. m.	Noon	2-3 p.m., time of minimum refraction	4-30 p.m.	8 a. m.	10 a. m.	Noon	2-3 p.m., time of minimum refraction	4-30 p.m.
	<i>feet</i>	<i>feet</i>	<i>feet</i>	<i>feet</i>	<i>feet</i>					
April, May 1906	2981.1	2981.3	2981.5	2981.0	2981.3	0.050	0.049	0.049	0.045	0.048
April, May 1907	2981.3	2981.3	2981.0	2981.2	2981.4	0.054	0.051	0.052	0.051	0.053
March, May 1908	2981.1	2981.3	2981.6	2981.4	2981.9	0.053	0.052	0.052	0.051	0.054
October 1908	2981.3	2980.9	2981.0	2980.8	2980.6	0.080	0.074	0.072	0.073	0.077
April, May 1909	2981.1	2980.7	2980.9	2981.1	2981.7	0.057	0.053	0.053	0.052	0.058
Mean	2981.2	2981.1	2981.2	2981.1	2981.4	0.059	0.056	0.056	0.054	0.058

The resulting differences in height are very accordant throughout this period, in fact if we look at the mean results, there is practically no diurnal variation; the refraction varies somewhat, but the heights are not affected by these changes. There was a sudden jump in the refraction co-efficient for the observations taken in October 1908, which is unaccountable.

Looking at the results of this table, we may assume that when the rays of light between two stations pass at a considerable elevation above the surface of the ground, reciprocal observations may be taken at all hours of the day between 8 a. m. to 4-30 p. m., subject to a distance limit of say 15 miles. In this case the distance between the two stations is 10 miles. As was mentioned before, Mussooree and Nag Tiba are hill stations, the height of the former is 6930 feet and of the latter 9912 feet. For want of data we cannot say whether this assumption would hold good for elevated plateaux or plains of 7000 feet and upwards; we may find in future that an elevated plain of 7000 feet offers the same difficulties as a low level plain and that the absence of variation between Mussooree and Nag Tiba is due not so much to their elevation as to their pyramidal form.

9. *Table F.*—showing deduced heights from observations taken at Mussooree at time of minimum refraction (2 p. m. to 3 p. m.)

Station observed	Distance	Refraction co-efficient	Year	Autumn	Spring	Autumn minus Spring
	<i>miles</i>			<i>feet</i>	<i>feet</i>	<i>feet</i>
Nojli (Shaw's) ...	45.927	0.069	1905-06	886.9	873.4	+13.5
			1906-07	886.6	875.3	+11.3
			1908-09	898.3	877.7	+20.6
			Mean	890.6	875.5	+15.1
Nojli T. S. upper mark ...	45.931	0.069	1906-07	937.7	923.3	+14.4
			1907-08	943.5	932.3	+11.2
			1908-09	949.3	928.2	+21.1
			Mean	943.5	927.9	+15.6
Bandarpunch Snow Peak ...	47.129	0.063	1905-06	20731	20713	+18
			1906-07	20724	20713	+11
			1907-08	20729	20715	+14
			1908-09	20740	20721	+19
			Mean	20731	20716	+16

It is interesting to note from the above table that the change from autumn to spring in the height of the plain station and also the snow peak computed from Mussooree is of the same sign and of practically the same amount. Since Mussooree, the station of observation, lies nearly midway between Bandarpunch and Nojli, the equality of the change seems to show that this seasonal variation is due entirely to change in refraction: and that the height of the snow peak has not been changed appreciably by snow fall.

Table G.—showing the variation in difference of height between 8 a.m. and 2 p.m. to 3 p.m. (time of minimum refraction) from observations taken at Mussooree.

Month and year of observation	Observed Station	Co-efficient of refraction	Deduced difference of height Observed station <i>minus</i> Mussooree		Difference in height deduced from 8 a.m. and 2 to 3 p.m. observations
			8 a. m.	2 p. m. to 3 p. m.	
November ... 1905	Bandarpunch	0.063	<i>feet</i> + 13799	<i>feet</i> + 13799	<i>feet</i> 0
April ... 1906			13789	13783	+ 6
November ... 1906			13797	13793	+ 4
April ... 1907			13792	13782	+ 10
Octr. and Novr. 1907			13805	13799	+ 6
March and April 1908			13803	13786	+ 17
October ... 1908			13816	13808	+ 8
April ... 1909			13798	13791	+ 7
		Mean	+ 13800	+ 13793	+ 7
November ... 1905	Srikanta	0.063	+ 13195	+ 13188	+ 7
April ... 1906			13176	13160	+ 16
November ... 1906			13193	13181	+ 12
April ... 1907			13192	13173	+ 19
Octr. and Novr. 1907			13206	13194	+ 12
March and April 1908			13192	13176	+ 16
October ... 1908			13220	13206	+ 14
April ... 1909			13195	13179	+ 16
		Mean	+ 13196	+ 13182	+ 14
November ... 1905	Jaonli	0.063	+ 14807	+ 14813	- 6*
April ... 1906			14787	14785	+ 2
November ... 1906			14807	14800	+ 7
April ... 1907			14808	14797	+ 11
Octr. and Novr. 1907			14817	14805	+ 12
March and April 1908			14816	14795	+ 21
October ... 1908			14840	14827	+ 13
April ... 1909			14820	14800	+ 20
		Mean	+ 14813	+ 14803	+ 10

* The refraction was less at 8 a.m. than at time of "minimum refraction."

Table G.—showing the variation in difference of height between 8 a.m. and 2 p.m. to 3 p.m. (time of minimum refraction) from observations taken at Mussooree.—(Continued).

Month and year of observation	Observed Station	Co-efficient of refraction	Deduced difference of height Observed station minus Mussooree		Difference in height deduced from 8 a.m. and 2 to 3 p.m. observations
			8 a. m.	2 p. m. to 3 p. m.	
November ... 1905	Kedarnath	0.063	<i>feet</i> +15919	<i>feet</i> +15931	<i>feet</i> —12*
November ... 1906			15925	15916	+ 9
October and Novr. 1907			15941	15926	+15
March ... 1908			15938	15915	+23
October ... 1908			15964	15947	+17
April ... 1909			15938	15913	+25
		Mean	+15938	+15925	+13
November ... 1905	Nojli (Shaw's)	0.069	—6022.9	—6043.0	+20.1
April ... 1906			6033.8	6056.5	+22.7
November ... 1906			6028.0	6043.3	+15.3
April ... 1907			6017.2	6053.1	+35.9
March and April 1908			6025.2	6054.2	+29.0
October ... 1908			6001.1	6031.9	+30.8
April ... 1909			6032.0	6052.2	+20.2
		Mean	—6022.9	—6047.7	+24.8
April ... 1906	Nojli T. S. upper mark	0.069	—5984.9	—6008.8	+23.9
November ... 1906			5976.1	5992.2	+16.1
April ... 1907			5978.6	6006.7	+28.1
November ... 1907			5943.6	5986.4	+42.8
March and April 1908			5969.9	5997.6	+27.7
October ... 1908			5952.0	5980.8	+28.8
April ... 1909			5980.3	6001.7	+21.4
		Mean	—5969.3	—5996.3	+27.0

* The refraction was less at 8 a.m. than at time of "minimum refraction."

Table H.—showing the variation in difference of height between 8 a.m. and 2 p.m. to 3 p.m. (time of minimum refraction) from observations taken at Nojli (Shaw's Station).

Month and year of observation	Observed station	Co-efficient of refraction	Deduced difference of height Observed station minus Nojli		Difference in height deduced from 8 a.m. and 2 to 3 p.m. observations
			8 a. m.	2 p. m. to 3 p. m.	
December ... 1905	Mussooree (Shaw's)	0.072	<i>feet</i>	<i>feet</i>	<i>feet</i>
March ... 1906			+6205.6	+6043.9	+161.7
December ... 1906			6127.4	6035.8	+ 91.6
March ... 1907			6185.6	6039.6	+146.0
January ... 1909			6123.7	6036.6	+ 87.1
March ... 1909			6190.2	6064.9	+125.3
			6141.6	6032.4	+109.2
		Mean	+6162.4	+6042.2	+120.2
December ... 1905	Bandarpunch	0.069	+20124	+19860	+264
March ... 1906			19958	19810	+148
December ... 1906			20124	19881	+243
January ... 1909			20098	19912	+186
		Mean	+20076	+19866	+210
March ... 1907	Srikanta	0.069	+19420	+19320	+100
January ... 1909			19535	19297	+238
		Mean	+19478	+19309	+169
March ... 1906	Jaonli	0.069	+20962	+20816	+146
January ... 1909	Kedarnath	0.069	+22286	+22059	+227

Tables G and H show the variation in height of the same station deduced from observations taken at 8 a. m. and at the time of minimum refraction respectively, and the following conclusions may be drawn from them :—

- (a) That the 8 a. m. heights are generally higher than the minimum time heights.
- (b) That the excess of the 8 a. m. over the minimum time heights is considerably greater when observations are taken from a hill station to a station in the plains than to another hill station, although the two stations observed may be practically at the same distance from the station of observation.
- (c) That the variation is very much greater and rapidly increases with the distance, when the station of observation is in the plains.

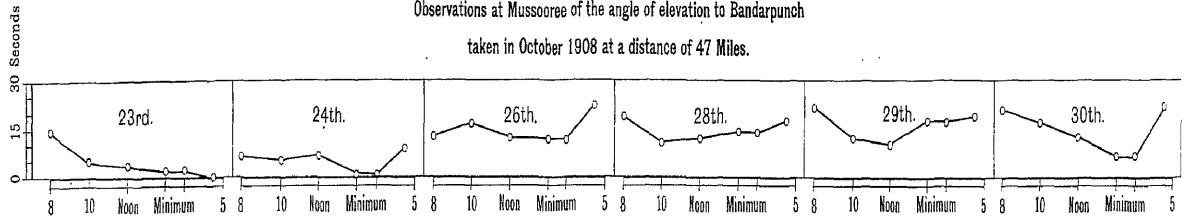
9. The results of the fifth, sixth and seventh series of observations have already been discussed in the preface to the G. T. Survey Synoptical Volume XXXV, pages ix to xiv.

10. The accompanying diagram illustrates how very great the differences are in the angles of elevation between morning and afternoon when observations are taken from a station in the plains to a hill station, and how very small the changes are between the morning and afternoon angles when observations are taken from one hill station to another.

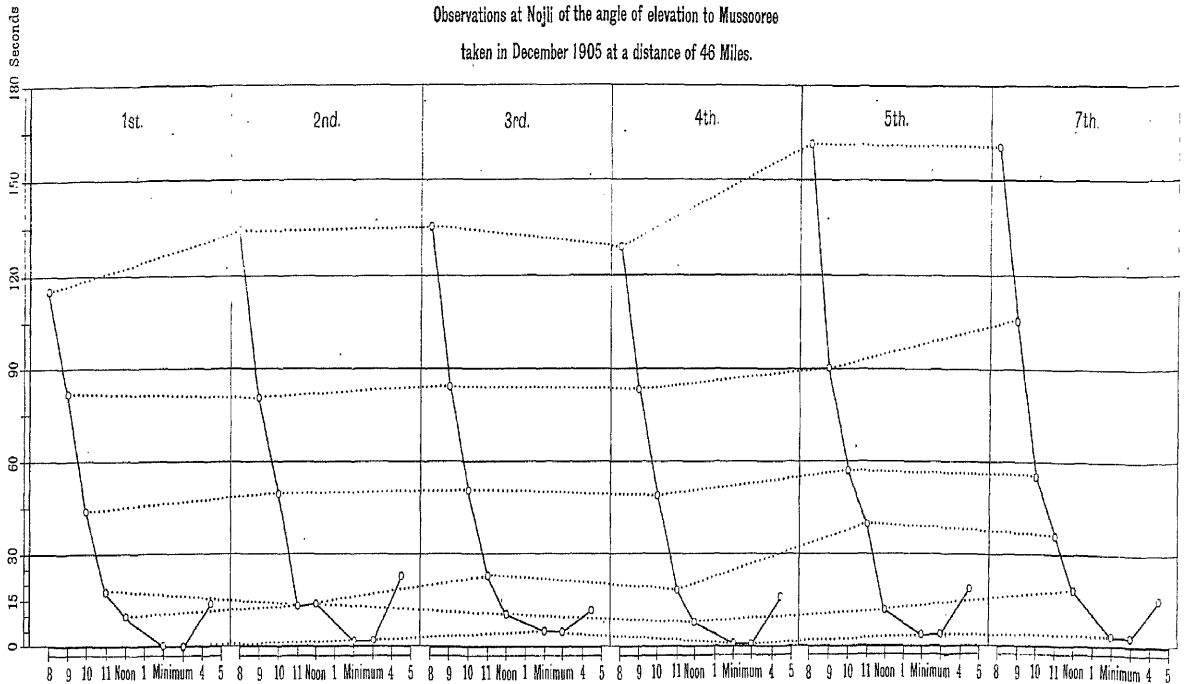
Nojli is in the plains, Mussooree is on the outer Himalayas and Bandarpunch is a snow peak of the main range (Kumaun Himalayas).

DIAGRAM SHOWING THE HOURLY AND DAILY VARIATIONS OBSERVED IN THE ANGLE OF ELEVATION.

Observations at Mussooree of the angle of elevation to Bandarpunch
taken in October 1908 at a distance of 47 Miles.



Observations at Naji of the angle of elevation to Mussooree
taken in December 1905 at a distance of 46 Miles.



The chain dotted lines show the variation in the angle from day to day when observed at the same time of day.

The continuous lines show the variation in the angle observed between 8 AM and 5 PM.